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AN EXPANDED ALTITUDE ALGORITHM FOR COMPUTING ALTITUDE-DEPENDENT CORRECTED GEOMAGNETIC COORDINATES

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13. ABSTRACT *(Maximum 200 words)*

A revised algorithm covering an expanded range of altitudes is described for computing altitude dependent corrected geomagnetic (CGM) coordinates from geocentric coordinates (and, where it exists, the inverse) using spherical harmonics. The original version was based upon the IGRF 90 magnetic field model, and was recently upgraded using the IGRF 95 model. In common with the two previous versions, the revised algorithm uses a tenth order spherical harmonic fit to the direction cosines (a unit vector) in a suitably chosen intermediate, altitude adjusted coordinate system. The additional coordinate system is needed to avoid convergence problems associated with the discontinuity in the CGM latitude at the magnetic equator at non-zero altitude. In previous versions the altitude dependence was obtained by computing the spherical harmonic fits to the Geographic -> CGM computation (and inverse) at 0, 300 and 1200 km altitude. They used a quadratic fit to interpolate each coefficient, and were limited to computing CGM coordinates and their inverse for an altitude range of 0 - 2000 km. In the revised algorithm the altitude dependence is based upon a fourth order polynomial fit of the spherical harmonic coefficients for 24 altitudes in the range of 0 - 7200 km. This provides an improved representation of the CGM compression around the South Atlantic Anomaly in addition to improved modeling of the increasing discontinuity with altitude at the magnetic equator. Comparisons are provided with previous approaches. Accuracy limitations and consistency between the direct and inverse computations are also discussed. Since magnetic field line tracing is the underlying basis, the revised algorithm provides an excellent alternative to field line tracing within the 0-7200 km range.

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Corrected geomagnetic coordinates, Coordinate conversions, Geographic coordinates, Geomagnetic latitude, Geomagnetic longitude, Altitude dependent coordinates, Gustafsson coordinates, International Geomagnetic Reference Field, Magnetic field line tracing

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The CGLALO95 line traced tables were generated using program GEOCGM which is due to the Papitashvili's and Gustafsson.

Bill McNeil of Radex tailored many special line tracing routines to meet the accuracy needs of this development.

1. INTRODUCTION

Since many ionospheric processes and the motion of charged particles in the ionosphere and the radiation belts are determined by Earth's magnetic field, it is desirable to use the geomagnetic domain for correlation with observations. For near Earth applications, the geomagnetic locations are usually expressed in Earth fixed geocentric coordinates, using a simple 6371.2 km radius "spherical Earth" model, which differs slightly in altitude and latitude from the usual geodetic "oblate Earth" model. In the early period of satellite exploration of the magnetosphere, the use of a coordinate system, geomagnetic coordinates, based upon a centered but tilted dipole representation of Earth's magnetic field was often sufficient for most applications. As more precise measurements became possible, the need arose for a coordinate system which would more closely represent the actual magnetic field. In 1958 Hultqvist published two papers [*Hultqvist*, 1958a; 1958b] defining a corrected magnetic coordinate system taking into account higher order terms in the spherical harmonic expansion of the 1945 magnetic field model. A real field line from Earth's surface may be traced to the centered dipole equator. This point is next defined to be equivalent to a line trace along a centered dipole field. The latitude and longitude of the point in dipole coordinates are the desired "corrected" geomagnetic coordinates.

In 1965 Hakura used the higher order terms in the spherical harmonic expansion of Earth's magnetic field to compute tables and maps of corrected geomagnetic coordinates. Since the actual magnetic field changes with time, it is necessary to generate new tables and maps. *Gustafsson* [1970 and 1984] has provided revised corrected geomagnetic coordinates based upon the International Geomagnetic Reference Field Epoch 1965 and Epoch 1980 (or IGRF65 and IGRF80). The earlier paper defines a set of hypergeometric functions which may be used to compute the corrections for a spherical harmonic representation of the magnetic field of order up to 7. Line tracing with modern computers makes the analytic approach unnecessary while permitting the use of higher order spherical harmonic field representations. More recently *Gustafsson, et al.* [1992] have performed similar calculations for the IGRF 1990 magnetic field model. The corrected geomagnetic coordinates provided in the tables described above are for 0 km altitude. Using this definition of CGM, there are areas of Earth's surface where magnetic field line traces never reach the dipole equator plane. *Gustafsson, et al.* [1984] describe various interpolation methods to fill in these forbidden areas.

True corrected geomagnetic coordinates are defined only at ground level, but a method is needed to provide geomagnetic coordinates at all altitudes. This report describes the development and implementation of such a method, which properly should be called "altitude adjusted corrected geomagnetic coordinates". Introduction of new terminology appears unwarranted however, and the altitude dependent conversions as well are referred to as corrected geomagnetic coordinates. Since the same procedure of field line tracing to the Earth-centered dipole equator applies, all points along a field line (on one side of the magnetic equator) have the same corrected geomagnetic coordinates (CGM). In practice, for non-zero altitudes, the actual approach taken is to trace down to zero altitude, and to then look up conventional CGM coordinates and interpolate using the tables printed in the above references. For IGRF90 and IGRF95 the look up and interpolation procedure has been automated in routines which we shall refer to as CGLALO90 and CGLALO95.

For non-zero altitudes at or near the magnetic equator, the field lines trace down to higher geomagnetic latitudes. The higher the altitude, the greater is the separation of the foot of the field line from the dip equator, so that low CGM latitudes do not exist for non-zero altitudes, and a significant discontinuity in latitude is present.

In a recent paper, *Baker and Wing* [1989] describe an alternative method to compute corrected geomagnetic coordinates. In their method, the corrected geomagnetic coordinates (and the corresponding inverses) are computed by evaluation of functions for the X, Y, and Z components of a unit vector obtained from a fourth order spherical harmonic expansion. They first computed the coefficients for the X, Y, Z components of a unit vector in the magnetic dipole coordinates for both the forward and inverse transformations for altitudes 0, 150, 300 and 450 km. They developed an interpolation/extrapolation scheme for computing the spherical harmonic coefficients for altitudes from 0 to 2000 km altitude. Since they were primarily concerned with representing higher CGM latitudes, the equatorial problem described above was not taken into account. As a result, the features of the South Atlantic Anomaly and the equatorial region are not well represented in the Baker and Wing computation. In the latter, the spherical harmonic expansion computations were performed in dipole magnetic coordinates. The coordinate conversions between geographic and dipole coordinates were accomplished using rotation matrices.

An improved method to calculate the CGM coordinates and their inverses at altitudes from 0 to 2000 km has been described in Bhavnani and Hein (1994). This approach is similar in many respects to that of Baker and Wing, but provides an improved representation of the South Atlantic Anomaly region, and a solution to the equatorial discontinuity problem. Spherical harmonic fits to the direct and inverse transformations were performed for altitudes 0, 300 and 1200 km, and the altitude dependence of the spherical harmonic coefficients was expressed as a quadratic interpolation and extrapolation to the individual coefficients using altitude/1200 km as the independent variable. Since the quadratic fit is uniquely determined by the values at the three altitudes, the representation of the spherical harmonic coefficients by the fit at those altitudes is exact.

In that paper, we indicated that improvements to the algorithm used were desirable, and we suggested that the use of an offset dipole may provide better results. We have carefully examined the possibility of making such improvements both by using an offset dipole, and by developing an improved altitude adjustment algorithm with the expectation that we would be able to make further improvements in the South Atlantic Anomaly region and in the equatorial discontinuity problem. However, we found that these approaches required much more complex code to account for the singular nature of the South Atlantic Anomaly, with poorer results near the poles, than that obtained with the original code. We then abandoned the offset dipole idea, and instead, sought to make improvements in the original algorithm.

In this paper we report improvements in the method described in our original paper, and the successful expansion of the altitude range from 0 - 2000 km to 0 - 7200 km. To simplify the presentation, portions of the original Bhavnani and Hein paper are incorporated in this report.

In Section 2 we describe the methodology used for the solution of this problem. Although it is, in principle, possible to expand the altitude range further, it would probably be necessary to take into account the external field (ring currents) for altitudes greater than 7200 km.

In Section 3 we provide a detailed description of the computation of the spherical harmonic coefficients. In Section 4 we describe the results obtained using this method, together with graphs.

In Section 5 we describe the use of the revised algorithm for field line tracing. We provide a description of the accuracy, and limitations of the algorithm for this application. Maps (in Geographic Coordinates) displaying error ranges (maximum difference along a precise field line trace, and the use of the algorithm) are displayed for a uniform latitude/longitude grid, for field line traces from 7200 km to ground and 800 km to ground.

In Section 6 we provide conclusions, and some comments concerning the possibilities of improving the algorithm for line tracing applications.

2. METHODOLOGY

For the analysis of observations from satellites in circular or near circular orbits, the use of CGM tables and interpolation methods for a fixed altitude, such as found in CGLALO95, are well suited. For satellites in more eccentric orbits, and other applications at non-uniform altitudes, a functional representation in terms of a spherical harmonic expansion, such as implemented by Baker and Wing is more appropriate, because a single routine can inherently interpolate smoothly over the entire region of space of interest. The equatorial discontinuity problem is handled by using an auxiliary coordinate system (magnetic dipole coordinates at altitude) to compute the spherical harmonic coefficients which are incorporated into the code. A simple mapping is used to transform to and from dipole coordinates at altitude and dipole coordinates at 0 km altitude. The lengthier calculations involved are usually not a burden with modern computers, and computational efficiency techniques can be incorporated when working at a constant altitude.

In using a spherical harmonic representation of a function defined on a spherical surface, where the function is initially specified by a table of values, there must be sufficiently dense data in the table, and the order of the spherical harmonic expansion must be chosen to adequately represent the function at the tabulated values.

The spherical harmonic coefficients for a function f, $a_{l,m}$, are usually computed from the following integrals.

$$a_{l,m} = \int_{\Omega} f(\theta,\phi)\; Y_{l,m}(\theta,\phi)\; d\Omega \tag{1}$$

To compute these integrals it is necessary to have a completely defined uniform grid. We found that a table of values between -88 and 88 degrees latitude at 2 degree intervals, and 0 - 350 degrees in longitude at ten degree intervals is adequate for a tenth order spherical harmonic expansion. All the computations described here were made with such a coordinate grid; in the remainder of this report, such a grid will be referred to as a standard coordinate grid.

A significant aspect of spherical harmonic fitting is the problem of convergence. In the theory of Fourier series, there is a problem with the convergence of the partial sums of the Fourier expansion for a function in the vicinity of a discontinuity. A typical example is the case of a step function, in which the partial sums oscillate in the vicinity of the discontinuity. Similar, but less pronounced behavior, occurs when the function to be represented is continuous, but has a discontinuous first derivative.

To avoid this problem (Gibbs' phenomenon), the functions chosen for the spherical harmonic expansion must be a periodic function in longitude (or its equivalent) and have no discontinuities. For that reason it is not practical to use the longitude variable itself. The simplest reasonable choice of functions are the complex exponentials exp $i\theta$ and exp $i\phi$ (or their real two-dimensional vector equivalents) where θ and ϕ are the co-latitude and longitude respectively. However, using this approach, problems arise with the quality of the spherical harmonic expansion fit to the actual data near the magnetic poles. We choose the unit vector approach used by Baker and Wing because the spherical harmonic expansion fit does not exhibit any pathology in the vicinity of the magnetic poles.

Since CGM values for non-zero altitudes have a discontinuity near the magnetic equator, it is not practical to use a ground-based dipole coordinate system for either computing the spherical harmonic coefficients or the spherical harmonic expansion. We used an at-altitude dipole-coordinate system at each of the selected 24 altitudes between 0 and 7200 km to perform these computations. The altitude dependent mapping described above to transform between the actual CGM latitude λ_{CGM} and an at-altitude dipole latitude λ_{dipole} is given by:

$$\cos^2 \lambda_{dipole} = \left(1 + \frac{altitude[km]}{6371.2}\right) \cos^2 \lambda_{CGM} \tag{2}$$

and is identical to that used in previous versions of the code. The use of the at-altitude dipole coordinate system given by the above transformation (and its inverse) effectively "closes" the discontinuity, permitting the calculation of the spherical harmonic coefficients at the selected reference altitudes. Other improvements in the closure of the discontinuity were also made, and are described in Section 3.

To compute the spherical harmonic expansion at arbitrary altitudes between 0 and 7200 km, the 24 sets of spherical harmonic coefficients for the reference altitudes were fit to a fourth order polynomial fit (using altitude/7200 as the independent variable).

For the computation of the geocentric to CGM coordinates, the procedure used was as follows:

(1) Compute (if new altitude was different from the last) the altitude dependent spherical harmonic coefficients.

(2) Compute the spherical harmonic expansion for the X, Y, Z components of the unit vector describing the orientation of the transformed point in the altitude dependent dipole coordinate system.

(3) Compute latitude and longitude of point, and apply the altitude transformation to the at-altitude dipole latitude. Return the computed corrected latitude and longitude values.

For the inverse computation, the procedure used was as follows:

(1) Compute (if new altitude was different from the last) the altitude dependent spherical harmonic coefficients for the inverse calculation.

(2) Transform the CGM input latitude to the at-altitude dipole latitude. Set error return flag and default return value if input latitude was invalid.

(3) Compute the spherical harmonic expansion for the X, Y, Z components of the unit vector describing the orientation of the transformed point in geocentric coordinate system. Return the computed geocentric latitude and longitude values.

3. GENERATION OF THE SPHERICAL HARMONIC COEFFICIENTS

The spherical harmonic coefficients were computed for the components of a unit vector in the target coordinate systems, as defined by the direction cosines for each of the 24 selected altitudes. For each altitude, the coefficients were computed using the standard coordinate grid tables using the standard formulas for computing spherical harmonic coefficients.

3.1 GEOCENTRIC TO CGM TRANSFORMATION

For each geographic longitude in the grid, a code was used to compute the geographic latitude of the dip equator. This means that the dip equator becomes the actual equator for the CGM coordinate system. Note that the dip equator does not lie in a plane in Geographic coordinates.

For 0 km altitude, the GEOCGM code, which implements the Gustafsson et al. definition of Corrected Geomagnetic Coordinates, was used to produce a table of CGM coordinates corresponding to a standard geographic coordinate grid. From this table, the entries within a band of ~15 degrees around the magnetic dip equator were deleted, and replaced by values obtained using a spline fit, with the added constraint that the spline curve passes through the coordinates of the dip equator. The modified table was incorporated into IGRF 95 (CGLALO95) version of a FORTRAN subroutine CGLALO which uses a look-up table and interpolation procedure to compute CGM coordinates for arbitrary points at 0 km altitude.

For each of the 23 non-zero altitude values, the field lines for the IGRF 95 magnetic field model from altitude to ground were computed using a precise field line trace routine. CGLALO95 was then used to compute the CGM tables for the respective altitudes. For the expanded version of the code (0 - 7200 km) the altitude dependent latitude adjustment algorithm was applied, and a spline fit through the dip equator was performed for latitude in a similar manner as was used in the 0 km altitude case. This procedure resulted in a uniform width to the altitude-dependent gap, and consequently improved spherical harmonic coefficient fits, by eliminating "clumping" in the vicinity of the South Atlantic anomaly. The 23 resulting tables were used to generate the spherical harmonic coefficient fits for the selected altitudes. For each spherical harmonic coefficient a fourth order polynomial fits was computed using altitude/7200 km as the independent variable. In the fitting process, weighting of the various altitude terms were performed, in such a way that only small deviations were permitted in the 0 altitude terms, so as to improve the fit in the 0 - 1200 km region. The same set of weights were used for fitting both the forward and inverse coefficients.

3.2 CGM TO GEOCENTRIC TRANSFORMATION

For the inverse transformation, an inverse altitude dependent routine was written, based upon the CGLALO95 inverse routine. The original CGLALO95 inverse routine uses the CGLALO95 routine together with a Newton-Ralphson algorithm for computing the inverse. The new routine replaced the CGLALO95 routine by a routine which uses the altitude dependent fourth order polynomial fit of the direct spherical harmonic fit. For each of the 24 altitudes, the desired tables were computed for a standard grid of CGM coordinates using the new routine.

The resulting 24 altitude inverse tables were used to generate the required spherical harmonic expansion coefficients, and the latter used to compute the required inverse fourth order altitude dependent polynomial spherical harmonic fits.

4. DESCRIPTION OF THE RESULTS

The original version (based upon the IGRF 90 magnetic field model) of the new code provided a substantial improvement in the representation of the equatorial region, while retaining excellent agreement with the Gustafsson tables (generated by the modified GEOCGM code) at the poles and at medium latitudes. Figures 1 and 2 from the original report [Bhavnani and Hein (1994)] are reproduced here for the convenience of the reader. Figure 1 is a graph of the CGM coordinates from the corrected Gustafsson et al [1992] IGRF 1990 model tables. Figure 2 provides a similar graph from the Baker/Wing code which was also computed from similar Gustaffson's tables for epoch 1987. Figures 3, 4 and 5 are graphs of CGM coordinates at 0, 800 and 7200 km altitude obtained from the expanded version of the IGRF 95 model code. In Figures 4 and 5 there is a marked bending of the constant geographic longitude curves at the edge of the equatorial gap for certain longitudes. This bending is not an artifact, but reflects the CGM longitude variation of the lines of constant geographic longitude in the vicinity of the dip equator.

Ideally, the output of the Geocentric to CGM calculations, fed into the inverse computation, should reproduce the original input coordinate grid. This test of the consistency of the direct and inverse transformations is illustrated in Figures 6, 7 and 8 for 0, 800 and 7200 km altitude. These graphs exhibit some deviations from the uniform spacing of the original grid, particularly in the vicinity of the poles, and in the vicinity of the South Atlantic Anomaly.

Since longitude differences at the poles are less relevant than at the equator, a more accurate measure of the differences between the original coordinates, and those obtained from the consistency calculation is the great circle arc between the coordinate pairs. Tables 1 and 2 provide the fraction of table values which lie in the following error intervals (in degrees) for the Geocentric ==> CGM ==> Geocentric and (where they exist) the CGM ==> Geocentric ==> CGM coordinates respectively:

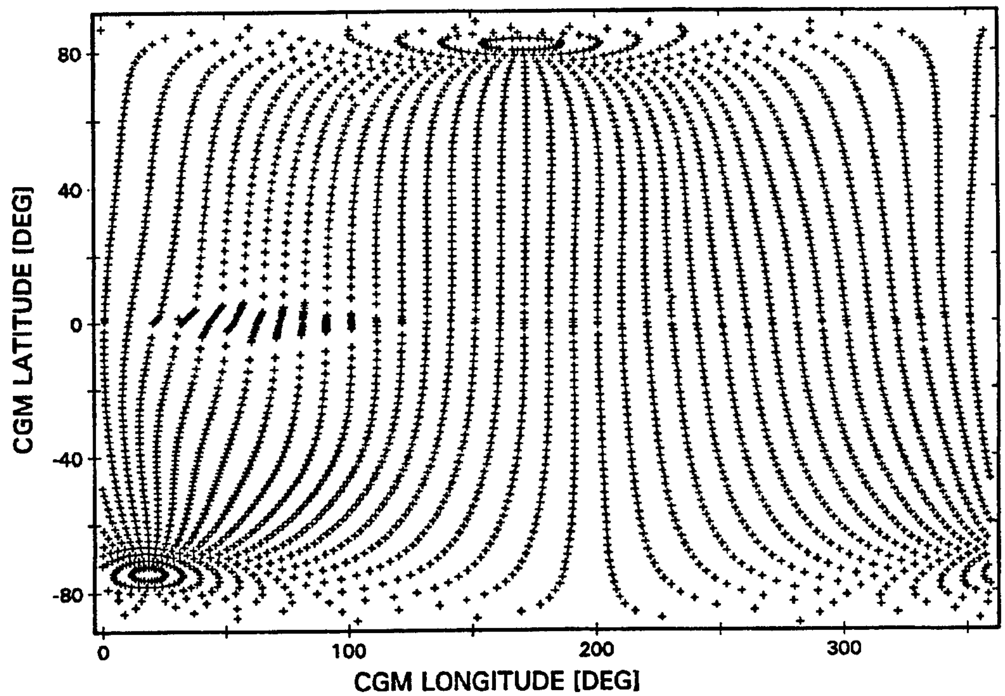

Figure 1. Corrected Geomagnetic Coordinate Grid from typical GEOCGM generated CGLALO tables for an array of CGM coordinates spaced in 2° in geographic latitude and 10° in geographic longitude.

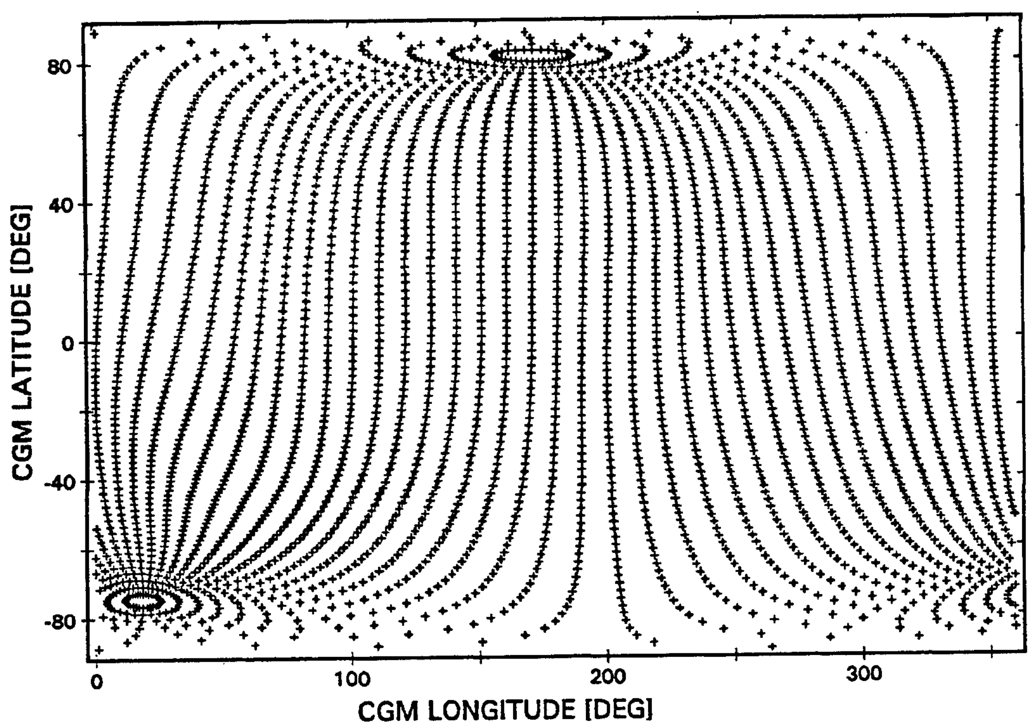

Figure 2. Same grid as Figure 1 for Baker-Wing Routine at 0 km altitude.

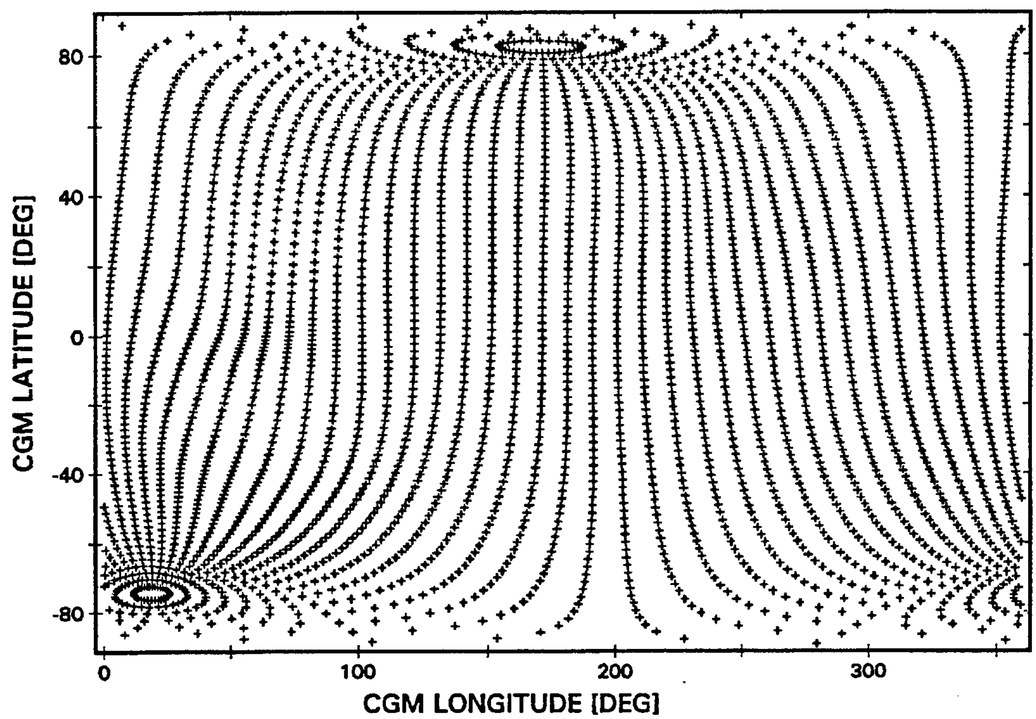

Figure 3. Grid for new routine, at 0 km altitude.

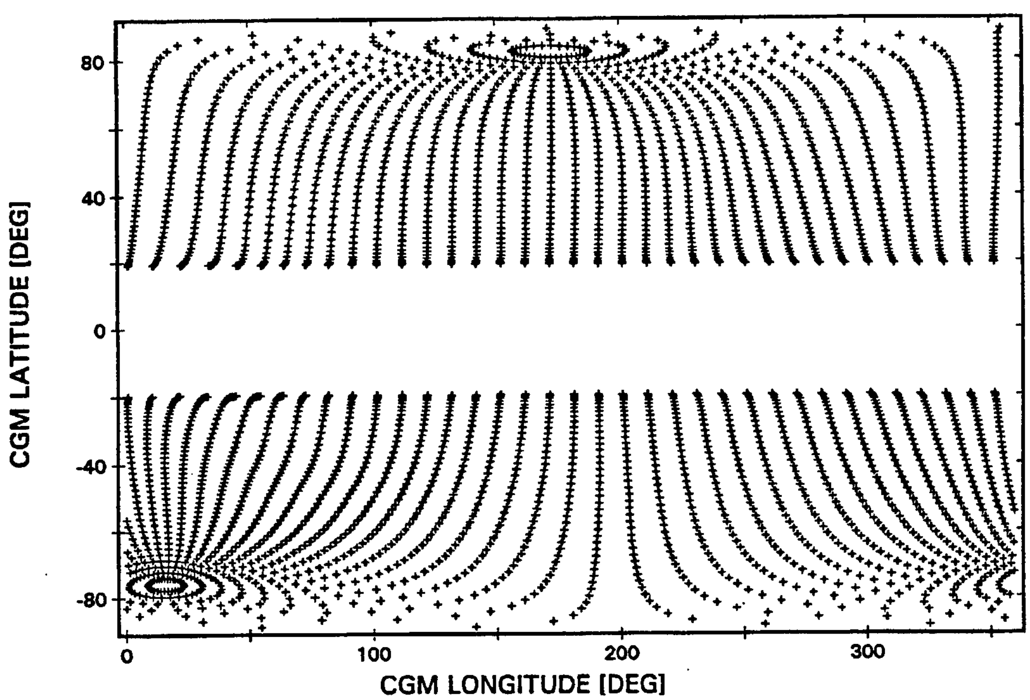

Figure 4. Same grid for new routine at 800 km altitude.

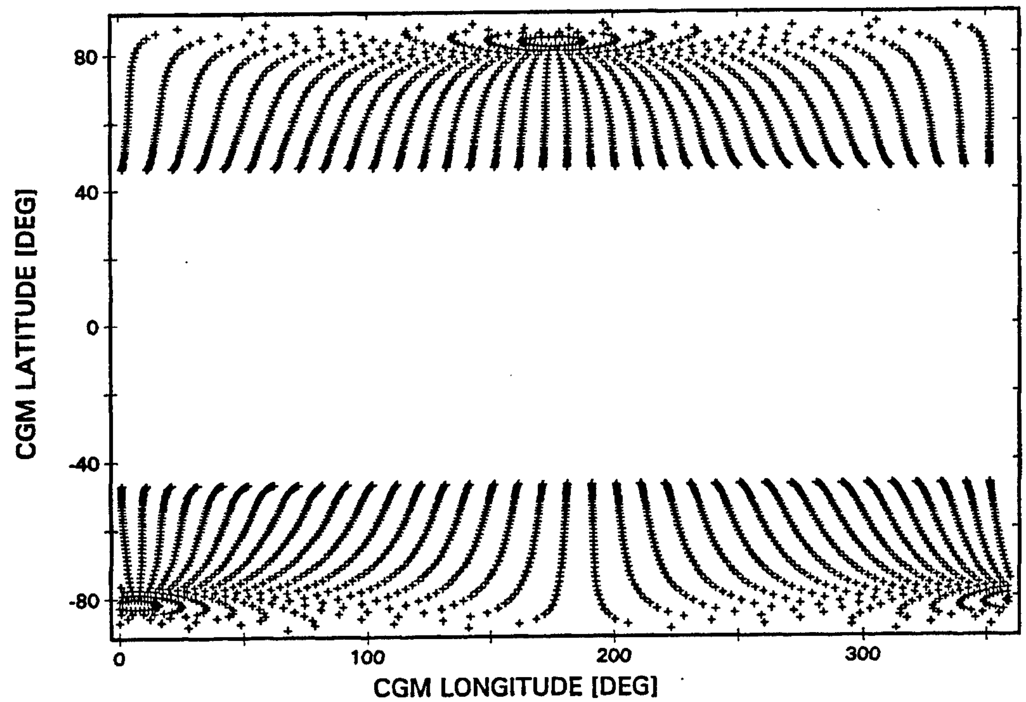

Figure 5. Same grid for new routine at 7200 km altitude.

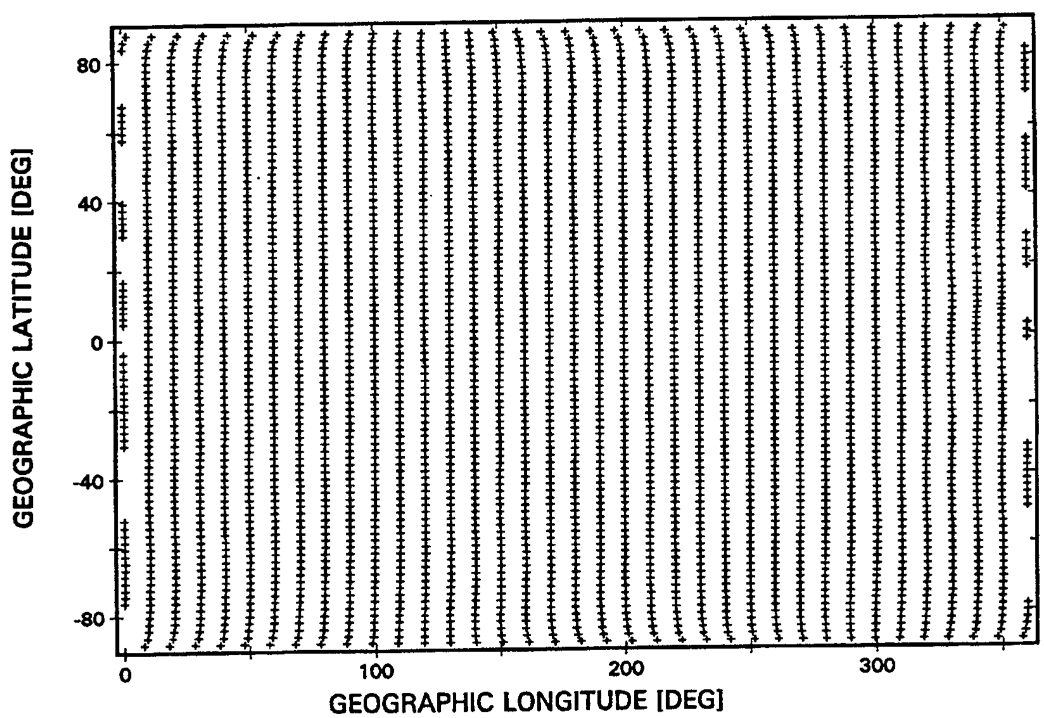

Figure 6. Same grid, showing consistency of direct and inverse conversion at 0 km altitude.

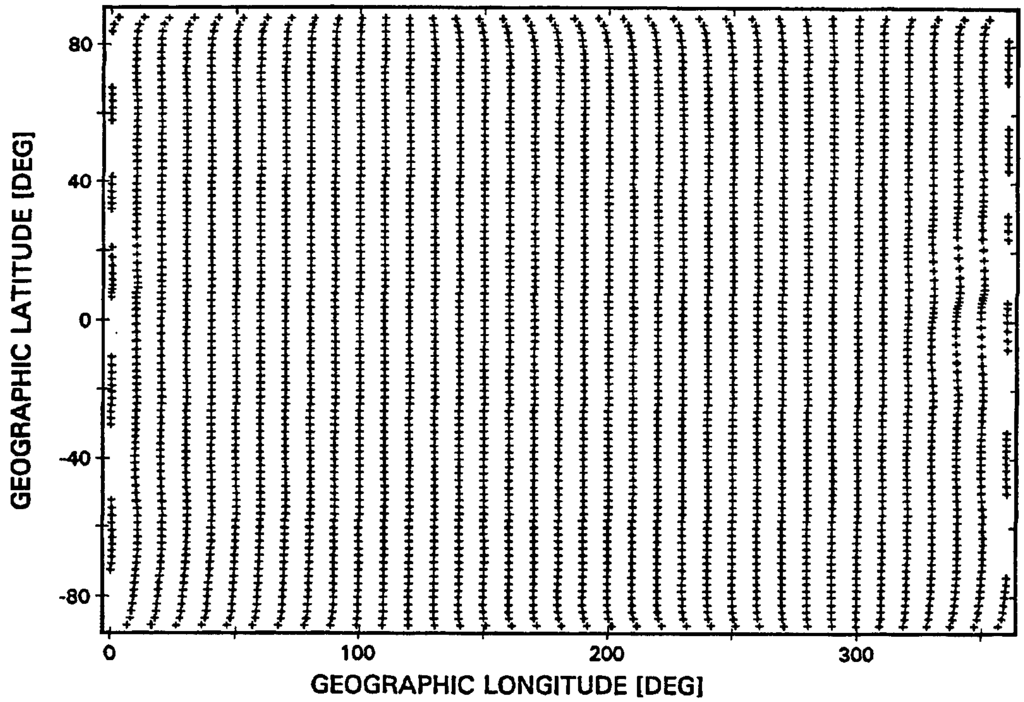

Figure 7. Grid showing consistency of direct and inverse conversion at 800 km altitude.

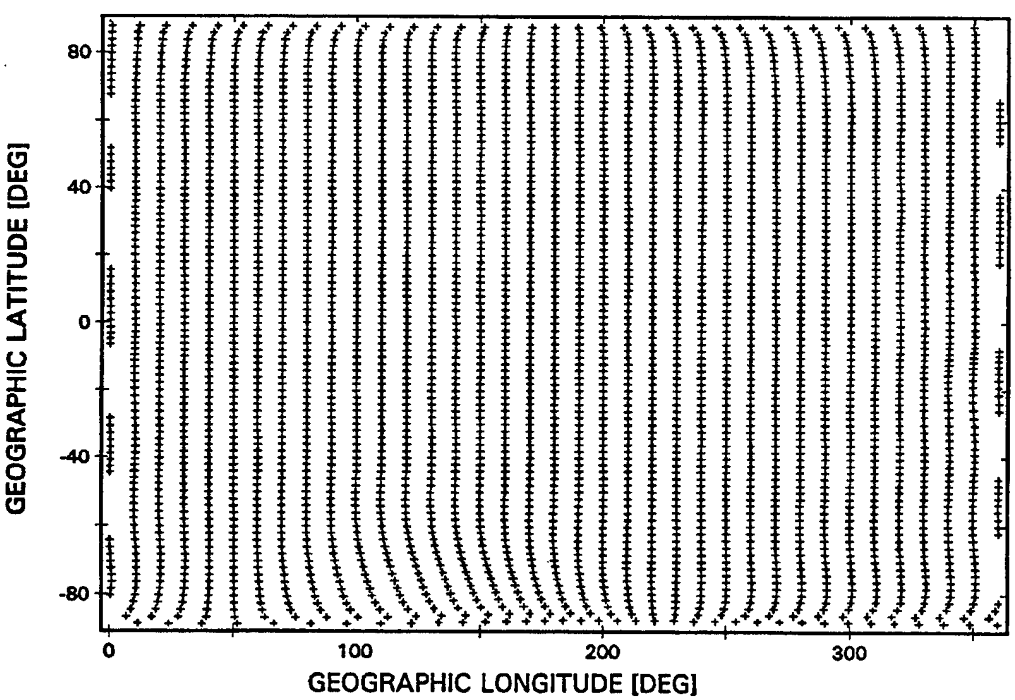

Figure 8. Grid showing consistency of direct and inverse conversion at 7200 km altitude.

0° - 0.1°, 0.1° - 0.2°, 0.2° - 0.5°, 0.5° - 1.0°, 1.0° - 2.0° and > 2.0°.

Thus, except for problems near the South Atlantic Anomaly, and near the "forbidden" band at altitude, the new algorithm performs adequately throughout the 0 - 7200 km altitude regime. Tests have been made for higher altitudes (extrapolating above 7200 km using the fourth order polynomial fits), with the finding that there is a rapid degradation of the Geographic --> CGM --> Geographic consistency calculations at altitudes above 8000 km.

TABLE 1. Inversion Error Analysis

GEOGRAPHIC == > CGM ==> GEOGRAPHIC

FRACTION OF ERRORS IN RANGE [deg]

ALT [km]	0.0-0.1	0.1-0.2	0.2-0.5	0.5-1.0	1.0-2.0	> 2.0
0	.75687	.17790	.04931	.01186	.00406	.00000
100	.73439	.17697	.06523	.01404	.00905	.00031
200	.69039	.19913	.07803	.02060	.00936	.00250
300	.66167	.21192	.08458	.02715	.00968	.00499
400	.64232	.22253	.08895	.02903	.00999	.00718
500	.63233	.21473	.10144	.03215	.01124	.00811
600	.62484	.21255	.10768	.03496	.01124	.00874
800	.61579	.20755	.11860	.03870	.01061	.00874
1000	.61517	.20724	.11923	.03995	.01155	.00687
1200	.62453	.20755	.11361	.03933	.01030	.00468
1600	.65418	.20787	.09800	.03246	.00749	.00000
2000	.68571	.20880	.08458	.01873	.00218	.00000
2500	.72878	.16948	.08177	.01561	.00437	.00000
3000	.69164	.15387	.12422	.02091	.00936	.00000
3500	.64669	.17915	.12921	.03246	.01217	.00031
4000	.64732	.17197	.12609	.04151	.01311	.00000
4500	.66011	.16698	.12422	.03964	.00905	.00000
5000	.64950	.20037	.10705	.04307	.00000	.00000
6000	.49938	.27372	.14950	.06117	.01623	.00000
7200	.54994	.23034	.12859	.06211	.02903	.00000

TABLE 2. Inversion Error Analysis

CGM ==> GEOGRAPHIC ==> CGM

FRACTION OF ERRORS IN RANGE [deg]
Note: Points for which the CGM input values were not valid were excluded.

ALT [km]	0.0-0.1	0.1-0.2	0.2-0.5	0.5-1.0	1.0-2.0	> 2.0
0	.78277	.16479	.03839	.01373	.00031	.00000
100	.76220	.17514	.04844	.01355	.00068	.00000
200	.74395	.18091	.05876	.01389	.00249	.00000
300	.73099	.18385	.06725	.01499	.00292	.00000
400	.71021	.19407	.07733	.01577	.00263	.00000
500	.71059	.18769	.08108	.01764	.00300	.00000
600	.69985	.19136	.08719	.01890	.00270	.00000
800	.69802	.19008	.09008	.02024	.00159	.00000
1000	.70057	.19404	.08619	.01879	.00041	.00000
1200	.70791	.19402	.08249	.01557	.00000	.00000
1600	.74866	.18638	.05959	.00538	.00000	.00000
2000	.79861	.16111	.04028	.00000	.00000	.00000
2500	.82589	.14633	.02778	.00000	.00000	.00000
3000	.79784	.13529	.06687	.00000	.00000	.00000
3500	.77244	.12500	.10203	.00053	.00000	.00000
4000	.76222	.12056	.11667	.00056	.00000	.00000
4500	.76910	.11458	.10127	.01505	.00000	.00000
5000	.79225	.10590	.06481	.03704	.00000	.00000
6000	.65341	.18939	.08965	.06755	.00000	.00000
7200	.68452	.17791	.06151	.06548	.01058	.00000

5. APPLICATION TO FIELD LINE TRACING

Field line tracing calculations are implicitly incorporated in the algorithm which generates altitude dependent spherical harmonic coefficients for the direct (Geographic - > CGM) transformations. Thus, although the algorithm was not developed for use as a field line tracing routine, it is possible to use it for that purpose, since, by definition, the CGM coordinates along a field line (in either the Northern or the Southern CGM hemisphere) are constant. Such use is limited by the accuracy of both the Geographic to CGM and inverse computations, and the consistency of both these transformations. The improvements in the consistency of the direct and inverse computations can be seen by comparing Tables 1 and 2 with similar tables in Bhavnani and Hein (1994). The revised algorithm exhibits a significant improvement for the purposes of field line tracing over the previous versions of the algorithm.

The use of the algorithm for field line tracing would proceed as follows:

1. Compute the CGM coordinates for the initial point (altitude, geographic latitude and longitude referenced to a spherical Earth).

2. Use the computed CGM latitude and longitude, and the desired altitude for the end point, as input to the inverse computation. Note, if the end point altitude is greater than that for the initial point, it is possible that the field line trace never reaches the end point altitude. The error flags must be checked to exclude this possibility. If the error flag is zero, then the computed Geographic latitude and longitude is, to the accuracy limits of the algorithm, the desired end point of the field line trace.

There are two ways of evaluating the performance of the revised algorithm for field line tracing:

1. Use a precise field line tracing routine to compute the geographic coordinates and altitudes along a field line, and then use the revised algorithm to compute the corresponding CGM coordinates for each point. Since the latter should be the same at each point on the field line, the maximum angular deviation (along a great circle arc) should be a suitable measure of the accuracy of the revised algorithm.

2. Use a precise field line computation to compare the field line traces with parallel computations using the revised algorithm as a substitute for field line tracing. Here, the great circle arc difference between corresponding points is a suitable measure of the accuracy.

A test of the first type was performed for a uniform grid, for line traces from 7200 km to the ground, and for 800 km to the ground. The maximum angular deviation (great circle arc) along each field line (field line traces were output at intervals of ~10 km in altitude). Maps which exhibit the regions in which the maximum deviation along a field line were greater than 0.2 degrees were also generated for the two cases described above. Different symbols are used to indicate different "bins" for the maximum deviation along the field line traces. Figures 9 and 10 are the maps for the 7200 km - 0 and 800 km - 0 cases.

The lack of a symbol at a grid point indicates that the maximum deviation along a field line beginning at the specified geographic coordinates and altitude and ending at the ground is less than 0.2 degrees. Otherwise, different symbols are used to indicate the range of maximum deviation, and, where it exists, "straddling". As expected, the worst behavior is exhibited along the CGM equator in the vicinity of the South Atlantic anomaly. Note that although the 800 km - 0 case performs better that the 7200 km - 0 case at higher CGM latitudes, its performance is worse (by a factor of ~3) in the South Atlantic Anomaly in the vicinity of the CGM equator. It should be expected that the performance of the line tracing comparison will degrade at lower altitudes in this region.

Tests of the second type were also performed, for line tracing from 0 km to various altitudes, with results similar to those already obtained. The worst behavior is found near the CGM equator in the vicinity of the South Atlantic Anomaly. Table 3 provides a summary of the results.

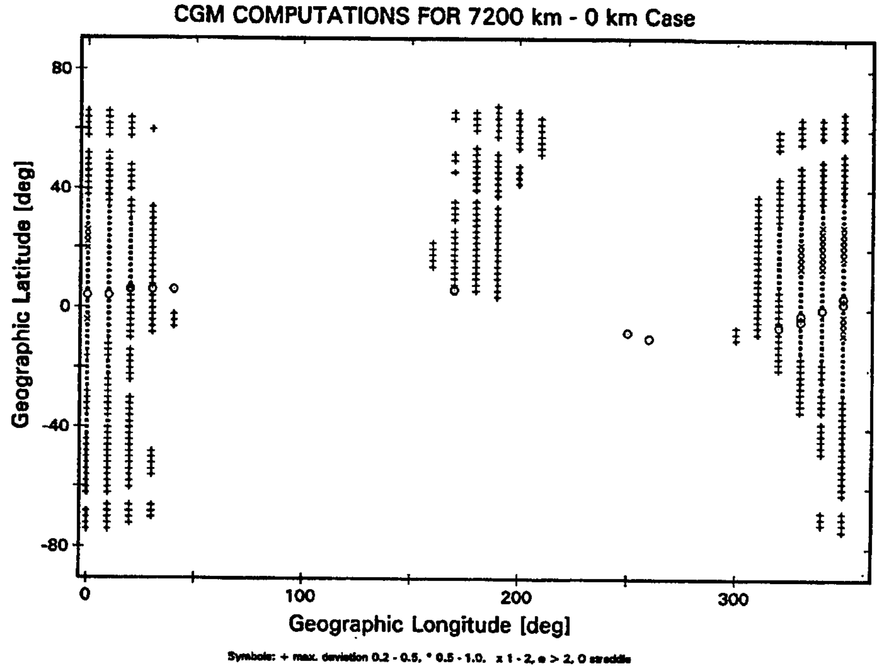

Figure 9. Map (standard grid, Geographic Coordinates) indicating grid points for which line traces from 7200 km altitude to ground exhibit a maximum deviation (great circle arc) > 0.2 degrees. The symbols used to indicate the range of maximum deviation: +: 0.2 - 0.5 deg, *: 0.5 - 1.0, x: 1.0 - 2.0, o: > 2, O: "straddle" points.

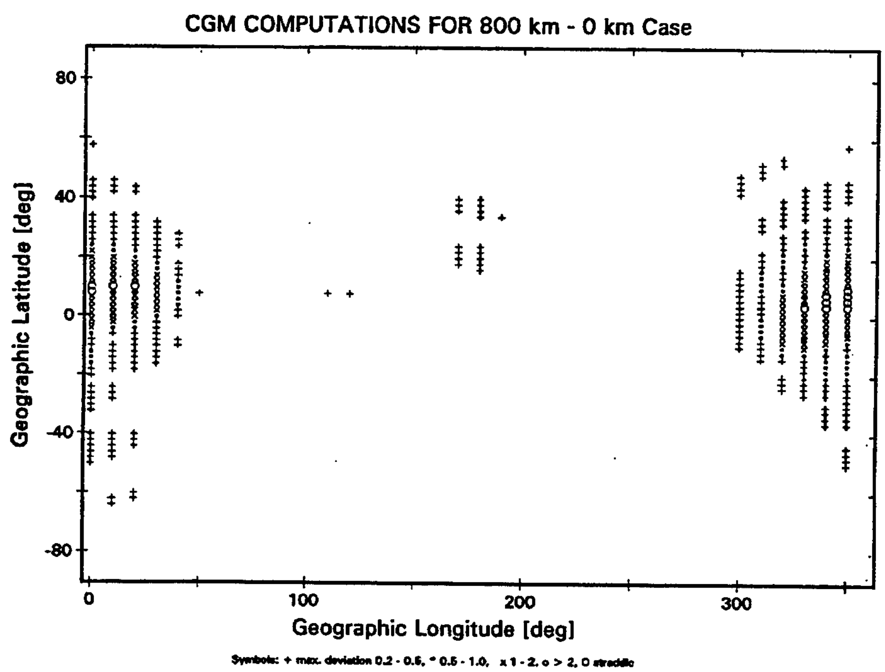

Figure 10. Map, same as for Figure 9, for field line traces from 800 km altitude to the ground.

TABLE 3. Comparison of Precise Line Trace Routine End Points with Those Generated Using Revised Algorithm.

Starting point of the line traces is 0 km, for points in a standard coordinate grid. Differences are in degrees (Geographic Coordinates) for a great circle arc. Note that the actual number of points for which the line trace exists at a given altitude decreases with altitude.

	Fraction of Data Points with differences in range					
Alt [km]	0.0-0.1	0.1-0.2	0.2-0.5	0.5-1.0	1.0-2.0	>2
400	.60521	.25996	.09907	.02309	.00633	.00633
800	.53707	.28537	.12265	.03848	.01242	.00401
1600	.51619	.28470	.14590	.03725	.01153	.00443
2400	.45240	.35721	.14856	.03029	.00817	.00337
3200	.36471	.40278	.20165	.01852	.00772	.00463
4000	.36200	.36418	.23517	.02722	.00653	.00490
4800	.33544	.38838	.22842	.03682	.00748	.00345
6000	.25155	.35167	.33127	.04326	.01607	.00618
7200	.27105	.29276	.35724	.05395	.01974	.00526

6. CONCLUSIONS

The IGRF 95 version of the CGM code developed by Bhavnani and Hein (1994) has been enhanced by extending the altitude range from 0 - 2000 km to 0 - 7200 km. The revised version retains the Cartesian spherical harmonic approach of *Baker and Wing* [1989] for geocentric to corrected geomagnetic conversion (and the inverse) and continues to make use of auxiliary coordinates (dipole coordinates at altitude) that are derived by applying a simple altitude adjustment algorithm to the CGM latitudes. In this auxiliary coordinate system the magnetic equator discontinuity described above is eliminated, permitting accurate fitting to 10th order spherical harmonic expansions. The revised version differs from previous versions by performing a fourth order polynomial fit to the spherical harmonic expansions generated at 24 fixed altitudes for both the direct and inverse transformation.

Corrected geomagnetic coordinates are linked to field line tracing, and are thus used extensively for particle mapping. There is also a need for a code to outline ionospheric effects at low latitudes. For this reason, altitudes of 100 and 200 km were included in the altitude array used to generate the revised IGRF 95 code (SFC95REV). Users who are interested in such possible applications are encouraged to use this revised code, and communicate with the authors, in order to develop future improvements of the code.

The use of the new algorithm for approximate field line tracing was discussed, and several tests have been described to examine the limitations of its use for this purpose. The results obtained indicate that, with the exception of a region near the CGM equator, in the vicinity of the South Atlantic Anomaly, the approximate results obtained may be suitable for many applications.

Here, it is important to note that the line trace capability, although implicit in the new algorithm, developed, not by design, but as a result of a desire to improve the original Kyle-Baker algorithm to better represent the transformation from Geographic to CGM coordinates near the CGM equator and lower CGM latitudes in general. Part of the software design constraints in the development of the new algorithm were to utilize as much of the original Kyle-Baker FORTRAN code as possible. The unexpected line tracing use, and its apparent success, albeit partial, suggests that the imposition of accurate field line tracing as a primary design requirement, without the above constraints, could result in a greatly improved and more useful algorithm.

Assuming the use of the same algorithm for application to IGRF2000 and beyond, and the same order spherical harmonic expansion, an update to the new CGM computation would only require the replacement of the existing sets of spherical harmonic expansion coefficients with new sets corresponding to the new magnetic field model for the direct and inverse transformations. The procedure for computing the new sets of coefficients was described in Section 3. In the FORTRAN code implementation of the new algorithm, this would be accomplished by replacing the existing BLOCKDATA section of the source code containing the expansion coefficients table with a new table.

7. REFERENCES

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APPENDIX: GLOSSARY AND NOTES

The Earth's magnetic field arises from contributions both within and external to it. For most near Earth applications (typically 1 Earth radius), the external field may be ignored. The internal field is described in terms of its geomagnetic potential, and is available in mathematical form as spherical harmonic coefficients and their secular variations. This model is the responsibility of the International Association of Geomagnetism and Aeronomy (IAGA), and is published periodically as revisions to the International Geomagnetic Reference Field (IGRF). Those more interested in the history and development of this subject are referred to the classic book *Geomagnetism* [*Chapman and Bartels*, 1940] which in turn describes the original work of Gauss, Schmidt who introduced Geomagnetic Coordinates, and many others.

Ionospheric phenomena near Earth are intimately controlled by the Earth fixed geomagnetic field and, because of the substantial and unnecessarily repetitive calculations involved in field line tracing, simplified models and procedures become essential. Although the dipole and the offset dipole models can be determined directly from the first and second order terms of the IGRF spherical harmonics and are useful for conceptual purposes, field line traces cannot be inferred with adequate accuracy from these models. Fortunately, the internal geomagnetic field is Earth fixed, and extensive a priori computations can be carried out to provide tables which relate geographic locations to their corresponding field line trace environment. This approach was used by *Hultqvist* [1958a, 1958b] to define and introduce Corrected Geomagnetic Coordinates, and subsequently revisions were made by *Hakura* [1965] and *Gustafsson* [1970]. Their work defines these coordinates with tables at the surface of Earth only. Later work leading to our present effort is described in the main text of this report.

Below we describe many of the terms covered or related to the present work. The asymmetrical nature of the geomagnetic field has given rise to the need for dipole, eccentric (or offset) dipole, corrected geomagnetic, and dip-pole representations, all of which are distinct in some manner. Thus, for instance, geomagnetic field lines are not truly perpendicular to Earth's surface at the corrected geomagnetic poles, but rather at the dip-poles. The reader should also be aware that the field undergoes a secular variation, and the assorted magnetic poles migrate one to a few kilometers per year.

Altitude Dependent Corrected Geomagnetic coordinates:
Extension of Corrected Geomagnetic Coordinates to altitudes above Earth's surface. Defined so that all points along a field line in the Northern or Southern hemisphere (in this paper, as defined by the dip equator) possess the same coordinates. Not part of Hultqvist's original definition of CGM coordinates.

Corrected Geomagnetic (CGM) coordinates:
Earth fixed magnetic latitude and longitude. Altitude is undefined. Prescribed by Hultqvist and Gustafsson and used in this report. Entails tracing along field lines to the dipole equator, and then determining the geomagnetic coordinates corresponding to this point on the dipole equator as if it had been reached by tracing along a pure dipole. Zero corrected geomagnetic longitude is the meridian which passes through the geographic South Pole, with East positive.

Corrected Geomagnetic (CGM) coordinate Poles:
Locations in the polar regions from where internal geomagnetic field line traces effectively intercept the dipole equatorial plane at an infinite distance. For Epoch 1990.0, the north and south corrected geomagnetic poles are at 81.0°N latitude and 278.5°E longitude, and at 74.0°S latitude and 126.0°E longitude, respectively. Inversely incidentally, the corrected geomagnetic coordinates of the geographic north and south poles are at 82.30°N latitude and 170.89°E longitude, and at 73.89°S latitude and 18.55°E longitude, respectively.

Dip Equator:
The plane at low latitudes where Earth's field becomes horizontal, so that the magnetic dip angle is zero. This resolves the problem with the Hultqvist procedure of tracing to the dipole equator, which results in imaginary latitudes when the field line terminates inside 1 Earth radius. Field lines undulate and the geographic latitude corresponding to zero CGM latitude sometimes had to be estimated by curve fitting.

Dip Poles:
North and south polar locations where the geomagnetic field at Earth's surface is vertical. Roughly at 78°N latitude and 256°E longitude, and at 65°S latitude and 139°E longitude, respectively.

Dipole:
Simple first order Earth centered representation of geomagnetic field. For epoch 1990, the first order X,Y,Z dipole moments are 1851, -5411, 29775 nanoTesla respectively, which places the dipole north magnetic pole roughly at 79°N latitude and 289°E longitude, and the south magnetic pole at 79°S latitude and 109°E longitude. The plane through the center of Earth normal to this axis determines the dipole equator.

Dipole Equator or Eccentric Dipole Equator:
Since the eccentric offset is roughly in the plane of the pure Dipole equator, the same equatorial plane through Earth's center, normal to the axis of the poles, applies to Dipole or to Eccentric Dipole. See *Dipole* and *Eccentric Dipole*.

Dipole Poles:
North and south intercepts of dipole axis with Earth's surface. See *Dipole*.

Eccentric (or offset) Dipole:
Field lines do not trace out normal to Earth's surface at the dipole poles, but considerably removed particularly for the South magnetic pole, which implies an eccentric rather than a centered dipole. The next few terms in the representation of Earth's field in IGRF 90 suggest an offset dipole centered approximately at geocentric X,Y,Z rectangular coordinates -400, 270, 190 km respectively, roughly in the direction of the Marianas Trench and farthest removed from the South Atlantic Anomaly.

Eccentric Dipole Poles:
North and south intercepts of eccentric dipole axis with Earth's surface, roughly at 82°N latitude and 259°E longitude, and at 75°S latitude and 119°E longitude, respectively. See *Eccentric Dipole*.

Geocentric:
Earth centered. Since latitude is defined by the angle between the vector to the location and the equatorial plane, geocentric coordinates imply a spherical Earth model. Magnetic field models, such as the International Geomagnetic Reference Field (IGRF) use 6371.2 km as the mean Earth radius to normalize radial distance and, for convenience, geocentric altitude refers to this radius when describing particle locations and the geomagnetic environment.

Geodetic:
Refers to oblate Earth and the local horizontal plane. Not used or implied in this report.

Geographic coordinates:
Earth fixed latitude, longitude, and altitude. Although commonly loosely applied to geodetic or geocentric, the spherical 6371.2 km radius of Earth applies throughout this report, with all latitudes, altitudes, and field line trace terminations determined by this model. Geodetic and geocentric longitudes are identical, with 0° passing through the Greenwich meridian, and East positive.

Geomagnetic coordinates:
Earth fixed magnetic dipole latitude and longitude. Altitude is undefined. The pure dipole axis is tilted with respect to Earth's axis and the poles approach the magnetic poles. Zero geomagnetic longitude is the geomagnetic meridian which passes through the geographic South Pole, with East positive.

Inverse Coordinate Conversion:
Obtains geographic coordinates, given CGM coordinates. The reverse of the geographic to CGM coordinate conversion. Since the conversions are altitude dependent, the altitude at which the geographic coordinates are desired must be specified. Except for the fitting approximations arising from analytical modeling, inversion following a coordinate conversion should return to the original latitude and longitude.